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Problem Solving in Mathematics and Scientific Reasoning

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Abstract

The paper describes the results of a study whose aim was to explore correlations among the components of the construct Culture of problem solving (mathematical intelligence, reading comprehension, creativity and ability to use existing knowledge) and six dimensions of Scientific reasoning, which was tested by the Lawson's Classroom Test. The total of 180 pupils from the Czech Republic aged 14–15 took part in this study. The results show that the dimensions proportional reasoning, control of variables and probability reasoning strongly correlate with the components mathematical intelligence, reading comprehension and ability to use existing knowledge.

Key words: *Culture of problem solving, Scientific reasoning, mathematics education, lower secondary school.*

1. Introduction

Problem solving, especially in mathematics, is still of significant interest in mathematics teaching and learning (Md Hassan & Rahman, 2017; Doulík, Eisenmann, Přibyl, & Škoda, 2016). This paper focuses on two constructs that are related to problem solving. The first construct is the modified *Culture of problem solving* (CPS), which was introduced by the authors of this paper as a tool for describing a pupil's preconditions for solving mathematical problems (Eisenmann, Novotná, Přibyl, & Břehovský, 2015). The other construct is *Scientific reasoning* (SR), which includes the thinking and reasoning skills that are involved in systematic exploration of a problem, formulating and testing hypotheses, evaluating experimental outcomes, etc. (Bao et al., 2009).

Science constitutes an important part of education in the area of STEM (*Science, Technology, Engineering and Math*) and it helps to develop SR efficiently. A number of researchers (e.g. Shayer & Adey, 1993; Bao et al., 2009) show that development of *science process skills* enables pupils to solve problems better (and this is well described by the structure of CPS for the area of mathematical problems) and to understand knowledge from the area of STEM. For example, Shayer and Adey (1993) show in their research that development of science process skills has a permanent impact on the general ability to learn. Similar conclusions in the research stimulated us to explore correlations between SR and CPS constructs.

Cihlář, Eisenmann, Hejnová, and Přibyl (2018) presented the results of a preliminary study conducted among 23 pupils aged 14–15 in the Czech Republic in 2016. More extensive research, the aim of which was to describe the mutual correlations between all components of the modified CPS (see section 2.1) and the SR dimensions, was conducted in 2017. The parameters of this new research allow us to accept its conclusions about these relations at standard level of significance. The paper presents results of this research.

2. Theoretical background

2.1 The Culture of problem solving

The composition of CPS is described in detail in Eisenmann et al. (2015). Let us stress that the CPS describes the conditions for success of an individual problem solver and is independent of the problem itself, and also of the solver's knowledge and their attitude to problem solving.

When originally developing the structure of the CPS, we primarily used the works of Schoenfeld, (1982); Sriraman, (2005); and Wu and Adams, (2006). "The problem-solving profile" (Wu & Adams, 2006) is understood as a tool for measuring a pupil's ability to solve problems. This tool focuses on two components that are also included in our CPS construct, namely: reading comprehension/extracting information from the wording of a problem and mathematical concepts, mathematisation of the problem and reasoning.

Originally, this form of CPS consisted of four components: intelligence, reading comprehension, creativity and ability to use the existing knowledge. The original component 'intelligence' has been replaced by a new component – *mathematical* *intelligence* (INTEL) in this study. This indicator was developed on the basis of works by Juter and Sriraman, (2011) and Gardner, (1993). Neither of the two tools was adopted as a whole. In both cases, only selected parts were adapted for our context (test administration, age of the respondents).

Reading comprehension (READ) is one of the competences that plays an important role in solving mathematical problems. Underdeveloped reading literacy is a significant obstacle, especially in the case of word problems (Vilenius-Tuohimaa, Aunola, & Nurmi, 2008; Fuentes, 1998; Pape, 2004).

In our original research, we work with creativity in the sense of divergent thinking (Guilford, 1967) and its relation to problem solving (Kwon, Park, & Park, 2006). By divergent thinking we mean the production of diverse but suitable answers to an open question or problem. We refer to this component as *creativity* (CREAT). Chamberlin and Moon (2005, p. 38) are convinced that higher creativity is prerequisite to the solution of non-routine problems in mathematics.

The *ability to use existing knowledge* (KNOW) is the fourth component of the CPS. This component has been developed by the team of authors with the aim of operationalizing the degree of formalism. This ability has been considered as prerequisite to successful solving of non-routine problems. Whilst solving such kinds of problems, knowledge itself is not sufficient; the solver must also be able to use it.

2.2 Scientific reasoning

Scientific reasoning can be characterised as a set of general skills that include specific thinking and logical processes, referred to as science process skills (Padilla, 1990). The American Association for the Advancement of Science (1989) formulated thirteen of these skills, which are divided into *basic science process skills* (observation, measurement, classification, quantification, inferring, predicting, identifying variables, communication) and *integrated science process skills* (interpreting data, controlling variables, operational definitions, hypothesizing, experimenting). This set of widely transferable skills reflects how scientists work, therefore it can help pupils to conduct scientific inquiry successfully. The individual dimensions of SR are not independent but create a certain hierarchy.

When conceptualizing Scientific reasoning, we, similarly to Lawson (1978), assumed that his structure stemmed from the hypothetical-deductive nature of science. That is why we base our research on an operational definition that enables us to assess scientific reasoning in six dimensions: conservation of matter and volume (CONSER), proportional reasoning (PROPOR), identification and control of variables (VARIABL), probability reasoning (PROBAB), correlational reasoning (CORREL) and hypothetical-deductive reasoning (HYPDED).

2.3 Correlations between CPS and SR

There is a whole range of aspects that play a role in the development of pupils' creativity, one of which is inquiry-based education (Kadir, Lucyana, & Satriwati, 2017). Inquiry-based education develops not only creativity, but also supports elimination of formalist thinking. Kotsari and Smyrnaiou (2017) state that this type of education eliminates formalism not only in the teaching of mathematics but also in the teaching of physics. Papáček (2010) states that creativity and a low level of formal knowledge are related to the level of pupils' science process skills. Since CPS includes the components CREAT and KNOW in its structure, we assume there must be correlations between the components of CPS and dimensions of SR.

3. Research questions and hypotheses

RQ: How do the individual components of the Culture of problem solving (CPS) correlate with scientific reasoning (SR)?

The first hypothesis arises from our preliminary research (Cihlář et al., 2018):

H1: The component KNOW from the CPS construct forms three pairs of dependent quantities
with dimensions of proportional reasoning, control of variables and probability reasoning of the SR construct.

The second hypothesis works with the component INTEL, which was not studied in the preliminary research:

H2: The component INTEL forms pairs of dependent quantities with all measured dimensions of the SR construct.

4. Methodology

The following subsections focus on the way of measuring both the constructs and the description of the research sample. One of the requirements of this study was that the research should be conducted using collective testing.

4.1 Culture of problem solving

All four CPS components were tested within a single 45-minute lesson. The parts of the test focusing on INTEL lasted 13 minutes, READ 13 minutes, CREAT 9 minutes and KNOW 9 minutes. All tested pupils were working independently, they were allowed to use only simple calculators. All parts of the test were evaluated by the authors of this paper.

The test of INTEL consisted of 8 problems. The problems could be divided according to the areas of study: logical reasoning (1 and 2); conception of infinity (3 and 6); spatial imagination – mental transformation (4); algebraic thinking (5); arithmetic patterns (7); geometric imagination in plane (8). All the test problems with the exception of problem 3 were closed multiple-choice tasks with one correct answer. The conceptual construct of the test was based on the following two principles: the first principle was the perspective of administering a test in a restricted time limit. If there were too many open questions, the testing would take long. This is connected to the other principle which was that the test should contain both open questions – represented by the subtest CREAT, and closed questions. What we tried to study in INTEL was not the pupils' creativity in the particular variables but the level of their sensitiveness to the above mentioned phenomena in individual areas. The test taker could get 2 points for each problem and the total sum indicates the index of INTEL. Problems 1 and 6 consisted of two questions, for each of which the test taker could get one point.

As far as READ is concerned, the pupils were set a short text of 15 lines. Afterwards, their task was to answer 4 closed and 2 open questions. The aggregate of all points formed the total score. The test was created on the same principle as used in the PISA research.

The level of CREAT was measured by Guilford's Alternative Uses Test. The pupils proposed as many 'uses of common objects' as possible. Qualitative evaluation of each test part was translated into points and the total score indicated an index of creativity. For example: The word "key" was presented to the pupils in the test. It was stressed out that what was meant was the key used for locking and unlocking. Alternatives that would get points were e.g.: hair decoration or gun in self-defence. Alternatives that were not rated: "treble clef" (mistaking object and word – the Czech word for treble clef includes the word key) or "to open the door" (this is not an alternative use).

The pupils' KNOW was assessed on the basis of a set of four problems. At the beginning of each problem, some item of previously learned knowledge was revised. This was followed by a simple application problem whose solution required active use of the particular item of knowledge.

4.2 Scientific reasoning

SR was tested by the *Lawson's Classroom Test of Scientific reasoning* (Lawson, 1978). We used the Czech version of the current version of Lawson's test released in the year 2000 (for shortened version see Dvořáková (2011); the full version was published as a part of dissertation of the author) and we carried out small corrections in items 8a and 8b according to Han, (2013).

The Lawson's test is a 24-item, two-tier test which involves a series of multiple-choice questions. Each of the two-tier items consists of a question with some possible answers followed by a second question giving possible reasons for the response to the first question.

The Lawson's test assesses pupils' reasoning abilities in the six dimensions mentioned above, including conservation of matter and volume (CONSER) (items 1 to 4), proportional reasoning (PROPOR) (items 5 to 8), control of variables (VARIABL) (items 9 to 14), probability reasoning (PROBAB) (items 15 to 18), correlation reasoning (CORREL) (items 19, 20) and hypothetical-deductive reasoning (HYPDED) (items 21 to 24). The items are of increasing difficulty.

When evaluating the test, a pupil would get two points for questions 1 to 22 if they chose *both* the correct answer to the question *and* the correct justification of the answer. The answers to the questions 23 and 24 were evaluated independently, that is the pupil got one point for each question answered correctly, or one point for selecting its correct justification. The pupils solved the test within a single 45-minute lesson.

4.3 Research sample

A total of 180 pupils (76 girls and 104 boys) aged 14–15 from one fourth grade of an eight-year secondary grammar school and from eight ninth grade classes from six lower secondary schools took part in our study. All schools were located in the Ústí nad Labem Region. In the research sample above average, average, and below average pupils were included.

4.4 Statistical evaluation

To assess the dependence or independence of individual quantities, Pearson's χ^2-test for contingency tables and Spearman's correlation coefficient were used. From multidimensional methods, cluster analysis and canonical correlation analysis were selected. The level of significance was used in all tests.

4.5 Preliminary research

The test of the component INTEL (the only test that was not validated sufficiently before our research) was developed at the beginning of 2017 in four rounds of pilot testing, always with about 40 pupils at the age of 14 to 15. Item analysis led to gradual selection of problems that were then used in the research study which is described here.

5. Results and discussion

24 pairs of quantities were studied, where the first quantity was one of the four components of CPS and the other quantity was one of the six dimensions of SR. Pearson's χ^2-test of dependence was conducted for all pairs of quantities of independence. Furthermore, Spearman's coefficient of rank correlation was examined and using cluster analysis the Euclidean distance between the corresponding standardized quantities was measured.

The seven strongest correlations between components of CPS and dimensions of SR are given in Table 1.

Table 1. Strongest correlations between components of CPS and dimensions of SR

Pairs of variables	χ^2-test			Spearman R	Distance
	χ^2	*df*	*p*-level		
READ & VARIABL	42.4741	9	0.000003	0.42134	12.9
INTEL & PROBAB	37.4531	6	0.000001	0.48732	12.4
KNOW & PROPOR	36.3491	8	0.000015	0.42078	13.6
INTEL & PROPOR	35.1819	6	0.000004	0.40103	13.0
KNOW & PROBAB	31.4128	8	0.000119	0.37935	13.9
KNOW & VARIABL	28.4091	12	0.004818	0.35383	14.4
READ & PROPOR	27.4276	6	0.000120	0.37116	13.9

It follows from the results in Table 1 that hypothesis H1 was confirmed.

When verifying the validity of hypothesis H2 it was found that the component INTEL makes pairs of dependent quantities with all the dimensions of SR, with the exception of HYPDED. The results from testing with the dimensions PROBAB and PROPOR are presented in Table 1, for the dimensions CONSER, VARIABL, and CORREL respectively the corresponding p-values are 0.002532, 0.022990, and 0.000451. However, the hypothesis on independence could not be rejected for the dimension HYPDED (). This means hypothesis H2 was not verified in its word-for-word form.

Canonical correlation analysis showed that the correlation coefficient of linear combinations of components of CPS and dimensions of SR with coefficients in

Table 2 was $R = 0.676$ ($F = 4.958, f = 24, p = 0.000$) and that this accounted for 49.3% of the variance. In further steps no other significant linear combinations were discovered.

Table 2. Coefficients of linear combinations with maximum correlation coefficient

CPS				SR					
READ	INTEL	KNOW	CREAT	CONSER	PROPOR	VARIA-BL	PROBAB	CORREL	HYPD-ED
0.426	0.491	0.313	0.103	0.226	0.259	0.208	0.300	0.349	0.021

The strongest dependences are shown in Figure 1.

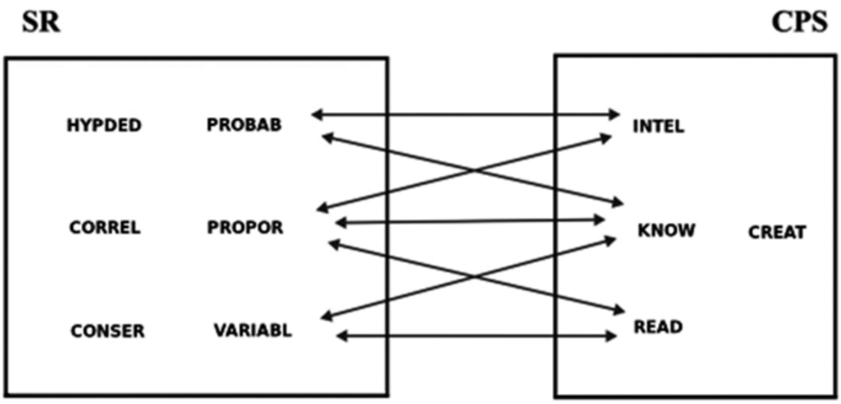

Figure 1. Strongest dependences between components of CPS and dimensions of SR

As stated above, the component KNOW of the CPS correlates strongly with three dimensions of the SR (PROPOR, VARIABL and PROBAB) (see hypothesis H1). In the following section we first interpret the dependences discovered.

PROPOR is a basic skill that children usually develop at a fairly young age. In learning, proportional reasoning is recognized as a fundamental reasoning construct necessary for mathematics and science achievement (Roth & Milkent, 1991). Since the component KNOW from the CPS was assessed on the basis of a set of the problems for some of which the skill of proportional reasoning was prerequisite, the dependence discovered between KNOW and PROPOR is one of the strongest.

The dimension VARIABL predominantly covers the skill to identify variables and the relations among them. It is a component of scientific inquiry, which is understood to mean skills in discovering or constructing knowledge for oneself (Dean & Kuhn, 2007). Similarly KNOW is prerequisite to successful solving of non-routine problems. That is why it is legitimate to expect the dependence between KNOW and VARIABL which was found in the research.

The problems in the Lawson's test targeting PROBAB are based on classical interpretation, that is the probability of an event is defined as the ratio of the number of outcomes favourable to the event, divided by the total number of possible outcomes. A precondition of correct solution of these problems is, among others, a good level of PROPOR. Taking into account what has been already stated above about the relation between KNOW and PROPOR, the stronger dependence between PROBAB and KNOW discovered is not surprising.

Similarly, the stronger dependence between the component READ of the CPS and the dimensions PROPOR and VARIABL of the SR comes as no surprise. A good level of skills needed to get information from a text is essential for successfully solving the problems in the Lawson's test that were used to diagnose the level of the dimensions PROPOR and VARIABL.

INTEL, in our interpretation, does not focus on testing specific knowledge and skills in mathematics but on a 'kind' of sense for mathematics. A strong dependence between INTEL and PROPOR and INTEL and PROBAB is not surprising. Both of these dimensions work to a certain degree with ratio and the sense of proportionality is developed in children first when learning and practicing operations in basic arithmetic. There is a weaker dependence between INTEL and CONSER, VABIABL and CORREL. These three dimensions of SR correspond to a certain degree with having a sense for mathematics. In mathematics lessons pupils learn about area and volume being maintained when geometrical objects are manipulated, which is the abstract foundation of the dimension CONSER. In algebraization of a solving process pupils get familiar with the basic ideas of variables, which are the background of VARIABL. At the age of 14–15 pupils have already been introduced to the functional approach to mathematics, which corresponds to the dimension CORREL.

It can also be stated that no correlation between INTEL and HYPDED was found. A possible reason for this may be that pupils may not be guided or encouraged to use combinatorial reasoning, and this is one of the possible topics for future research.

To conclude this section, let us briefly refer to the limits of this research study. Undoubtedly the scope of our research sample which consisted of pupils exclusively from one region was a limitation. However, we are convinced that this limitation has no major impact on the results presented here.

6. Conclusions

In our contribution we focused on the correlation between two constructs (CPS and SR) that explore the basic skills prerequisite for solving different types of problems. The discovered correlations and dependences were analysed in detail. The possible reasons for the correlations between these constructs were shown. We should emphasize that the individual dimensions of SR are not independent but create a hierarchy, which means that successful solution of tasks from a higher dimension supposes the mastering of tasks from the lower dimensions. The findings from this research show that three of the dimensions (PROPOR, VARIABL and PROBAB) correlate more significantly with KNOW, READ and INTEL. Thus we are convinced that the mastering of KNOW and READ by pupils at the end of lower secondary school is tightly linked with the development of more general skills. This is very important for school practice in particular, since it is clear that development of learners in the STEM area (e.g. in mathematics) is a good precondition for development of more general skills that are also applicable in other areas of education.

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