

Conclusions

Number sense and algebraic thinking work hand in hand as foundation for algebra courses. Symbol and pattern sense are the two most influential factors that affect the relationship between number sense and algebraic thinking. This information presents a new topic for discourse and confirms the importance of symbol sense and pattern sense in algebraic thinking development. Given that number sense and algebraic thinking are closely connected, symbol sense and pattern sense act as mediators between number sense and algebraic thinking in building strong foundation for future formal algebra learning. Symbol sense and pattern sense can intensively improve the conceptual understanding of underlying properties of algebra while learning arithmetic in elementary mathematics classrooms. The items shown in Figures 1 and 2 are the best examples on how teachers could encourage number sense and algebraic thinking. They could incorporate symbol sense and pattern sense in their daily lessons. Early algebraic thinking does not need a new chapter to include in the existing syllabus. It is how to teach arithmetic that can build a conceptual understanding and develop children's ability to think deeper beyond mere computations.

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