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Year Five Pupils' Number Sense and Algebraic Thinking: the Mediating Role of Symbol and Pattern Sense

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Abstract

This study mainly focused on the relationship between number sense and algebraic thinking. Previous studies have provided evidence that number sense plays an important role in developing algebraic thinking. The role of symbol and pattern sense are yet to discover in relation to number sense and algebraic thinking. The purpose of this study was to identify the mediating effects of symbol sense and pattern sense in year five pupils' relationship between number sense and algebraic thinking. To do so, two mathematics tests were carried out among 720 year five pupils in the district of Malacca, Malaysia. The collected data were analysed using a partial least squares-structural equation modeling approach. The data collected were analysed using SPSS 22.0 and SmartPLS 3.0. Results demonstrated that symbol sense and pattern sense are good mediators between year five pupils' number sense and algebraic thinking. This result of this study supports the past studies related to the role of number sense, symbol and pattern sense in developing algebraic thinking. The presented study provides suggestions as intervention to increase students' making sense ability in numbers, symbols and patterns to develop algebraic thinking.

Keywords: *early algebra, generalisation, patterns, sense making, symbols*

Introduction

In most school curricula, algebra is a formal equation comprising variables and signs for operations and equality. However, the underlying properties of algebraic thinking have always been neglected. To overcome this problem, previous studies have advocated algebrafying the elementary mathematics rather than confining algebra as a course to be taught in middle or high schools (Blanton & Kaput, 2003; Knuth, Alibali, McNeil, Weinberg & Stephens, 2011). With regard to this, studies have been carried out to develop algebraic thinking skills by carrying out activities such as working with patterns, arithmetic generalisation and importance of the equal sign (Carpenter, Franke, & Levi, 2003; Molina & Ambrose, 2008; Warren, Cooper & Lamb, 2006).

In general, arithmetic is taught at elementary school level. Often, arithmetic and algebra are treated as two different courses and there is no connection established while teaching arithmetic (Cai & Moyer, 2008; Herscovics & Linchevski, 1994). Nonetheless, the underlying properties of algebra could be developed with proper teaching instructions which encourage children's thinking beyond abstract calculations (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). The underlying properties of algebra have gained increased attention in recent years among mathematics researchers. Thus, it has created an awareness of algebraic thinking, which encompasses generalised arithmetic, modeling and functions (Kaput, 2008).

Similarly, other aspects of algebra such as making sense of numbers, working with patterns and conceptual understanding of the equal sign are crucial in early algebraic thinking (Carpenter, Levi, Berman, & Pligge, 2005; Kieran, 2004; Stephens, 2005). Looking into these aspects, previous studies have provided evidence that children's ability in number sense will enable them to build a conceptual understanding of relations involved in an algebra course (Carpenter et al., 2003). Studies have also been carried out of the roles of symbol sense and pattern sense towards the development of early algebraic thinking (Brizuela & Schliemann, 2004; Jacobs et al., 2007; Lannin, 2005; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011; Stacey, 1989).

Literature Review

In general, number sense refers to proficiency in mental calculation, computational estimation, judgment of the relative magnitude of numbers, recognition of part–whole relationships, and problem solving. It is one's conceptual understanding of numbers and operations, together with development of useful, flexible and efficient strategies for handling numerical problems (Yang, Hsu, & Huang, 2004). It is not merely focused on arithmetic calculations. According to Hsu, Yang and Li (2001), number sense comprises the following components; i) understanding number meanings and relationships, ii) recognizing the magnitude of numbers, iii) understanding the relative effect of operations on numbers, iv) developing computational strategies and being able to judge their reasonableness, and v) having the ability to represent numbers in multiple ways.

Number sense has also received attention in the discussion of the development of algebraic thinking. Number sense is an inevitable aspect which could lead to a smooth transition from arithmetic to algebra (Carpenter & Levi, 2000; National Council of Teachers of Mathematics, 2000). Number sense is algebraic in several ways. Thus, exposure to number sense could help the young learner to get a precise structural and algebraic understanding of numbers even before they learn to manipulate them (Strother, 2011). Blanton and Kaput (2003) asserted that algebrafying arithmetic activities helps children to do many things at once, including practicing number facts, developing number sense, and recognizing and building patterns to model situations.

Likewise, symbol sense also plays an important role in the development of early algebraic thinking (Brizuela & Schliemann, 2004; MacGregor & Stacey, 1997; Stephens, 2005). As algebra is all about working with symbols and equation, a good foundation in working with symbols could definitely create an effortless pathway to mastering formal algebra in middle school. Evidence shows that children are able to create their own kind of algebra when they generate general rules and exhibit these connections via symbols to represent operations and variables (Stephens, 2005). Hence, young students should be encouraged to make their own symbols inventions and not necessarily learn the algebraic formal notation (Berkman, 1998).

Another important aspect of symbols is the understanding of the equal sign. Elementary school children often perceive the equal sign operationally rather relationally (Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007; Rittle-Johnson et al., 2011). A relational understanding of the equal sign enables children to develop ideas of arithmetic representations. While an operational understanding of the equal sign could only enable them to master computational skills. Excellency in computational skills would not help them to build a conceptual understanding of underlying properties in arithmetic. This would hinder children from thinking in an algebraic way.

Last but not least, working with patterns has gained high attention among researchers of early algebraic thinking (Ferrini-Mundy, Lappan & Phillips, 1997;

Lannin, 2005; Lannin, Barker, & Townsend, 2006; Stacey, 1989; Warren et al., 2006). Looking for patterns in different situations, the use of symbols and variables that represent patterns and generalisations are crucial aspects of a conceptual understanding of early algebra. Using patterns is seen as way of approaching algebra (Mason, 1996). Patterning activities have been most suitable for children to express generalisation and create their own "rule" to find the subsequent terms in the series of patterns. This "rule" eventually develops into a function concept in later years of algebra learning. Fundamental elements of functions in algebra; input-process-output could be easily established and explained using patterning activities. This will nurture children with a conceptual understanding of functions.

The above-mentioned aspects such as number sense, symbol sense and pattern sense are associated with early algebra. However, to date no studies have been conducted to find if symbol sense and pattern sense could be good mediators between number sense and algebraic thinking. It is essential to look for this relationship to facilitate the improvement of teaching instructions and curricula documents. The essence of algebraic thinking development lies in identifying the right constructs to develop a conceptual understanding of algebra properties, which underlies arithmetic. With regard to this, the following research questions were posed:

1. Does symbol sense mediate the relationship between year five pupils' number sense and algebraic thinking?
2. Does pattern sense mediate the relationship between year five pupils' number sense and algebraic thinking?

In view of the above research questions, the following hypotheses were formulated to guide this study.

H1: Symbol sense mediates the relationship between year five pupils' number sense and algebraic thinking.

H2: Pattern sense mediates the relationship between year five pupils' number sense and algebraic thinking

Research Methodology

Research General Background

The study utilised a descriptive research design which is a cross-sectional study as the researchers collected data from a sample of a population identified in advance and carried out the study in a specific period of time.

Research Sample

The participants in the presented study were 720 year five pupils from national schools in the district of Malacca, Malaysia. A cluster random sample was used in this study, with students clustered by school. A list of national schools in that particular district was received from the Ministry of Education, Malaysia. Then, the researchers numbered those schools. With the help of the Rand() function available in Microsoft Excel, random numbers were generated. Then the names of the schools with respective numbers were chosen as a cluster. All year five pupils from those schools were involved in this study. The participants consisted of 370 (51.4%) female pupils and 350 (48.6%) male pupils. These pupils had not been exposed to any intervention previously. They had only had ordinary mathematics lessons conducted in schools.

Instrument and Procedures

Assessment of Number, Symbol and Pattern Senses (ANSPS) was performed to examine the year five pupils' number sense, symbol sense and pattern sense. Items in this assessment were adapted from literature. It comprised 16 items. All the items were multiple choice questions. These items were scored dichotomously: 1 for a correct response and 0 for an incorrect response. The items are arithmetic questions which examine the year five pupils' capability of making sense of numbers, working with simple symbols and numeric or figural pattern series. Figure 1 shows one of the items which tests number sense. By making sense of numbers, one could identify the answer by knowing that the product of 20 x 20 is 400. Since the multiplier and multiplicand are less than 20, the answer must be less than 400 but not too deviated from 400. This sense is essential to make sense of algebra in later years of education.

Which of the following is closest to the product of 18×19?

A. 250	B. 350
C. 450	D. 550

Figure 1. One of the number sense items

The second instrument was algebraic thinking diagnostic assessment (ATDA). This instrument was adapted from Ralston (2013). ATDA was selected because it was the only assessment tool available in the literature to measure elementary students' algebraic thinking which encompasses all three strands of algebraic thinking defined by Kaput (2008). ATDA consisted of 24 items which comprised generalised arithmetic, modelling and functions. All the items were short-answer questions. These items were also scored dichotomously: 1 for a correct response and 0 for an incorrect response. Figure 2 shows one of the items from algebraic thinking items. The 'c' in this item may not refer to the true meaning of a variable. However, the ability to work with this item shows that a pupil could think of relationships involved in addition and the equal sign. The understanding that 'c' refers to a common number enables the pupil to work accordingly.

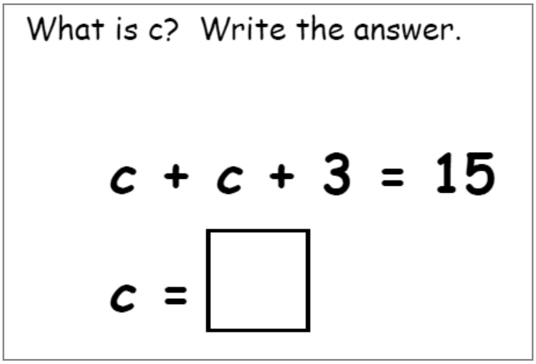

Figure 2. One of the algebraic thinking items

Data were collected with the use of two mathematics tests. Both tests were given on the same day to ensure the same students sit the tests. The first test was given before their break and the second one was given after the break. Each test lasted an hour.

Data Analysis

The study reports the results from part of a major study of algebraic thinking. The major study utilised structural equation modelling using the partial least square technique for data analysis. Hence, mediator analysis was used in Smart PLS. A construct acts as mediator when it intervenes between two other related constructs, as shown in Figure 3. In this case, Y2 acts as a mediator between Y1 and Y3. Baron and Kenny (1986) claimed that there are three necessary conditions that should be met in order to say that mediation exists. They are the following:

Y1 is significantly related to Y2.

Y2 is significantly related to Y3.

The relationship between Y1 and Y3 declines when Y3 is in the model.

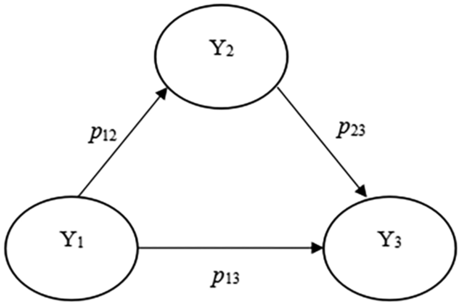

Figure 3. General mediator model

Y2 mediates the relationship between Y1 and Y3. The indirect effect is referred to the product of p12 and p23. The indirect effect should be tested for significance. To check the significance, bootstrapping procedure needs to be used. Indirect effects for 5000 samples should be calculated using Microsoft Excel. Subsequently, standard deviation is calculated based on these 5000 samples' indirect effect. The t value will be the indirect effect divided by bootstrapping standard deviation. If the indirect effect is significant, assessment of variance accounts for (VAF), calculation will take place in order to determine the mediation level. VAF can be calculated by the following formula:

$$VAF = \frac{p_{12} \cdot p_{23}}{(p_{12} \cdot p_{23} + p_{13})}$$

The VAF value of over 80% refers to full mediation, between 20% and 80% it is categorised into partial mediation and under 20% it is considered as no mediation.

Research Results

Figure 4 shows the model tested the mediating effect of symbol sense on the relationship between number sense and algebraic thinking, while Figure 5 shows the mediating effect of pattern sense on the relationship between number sense and algebraic thinking. Table 1 summarizes the significance of the test results.

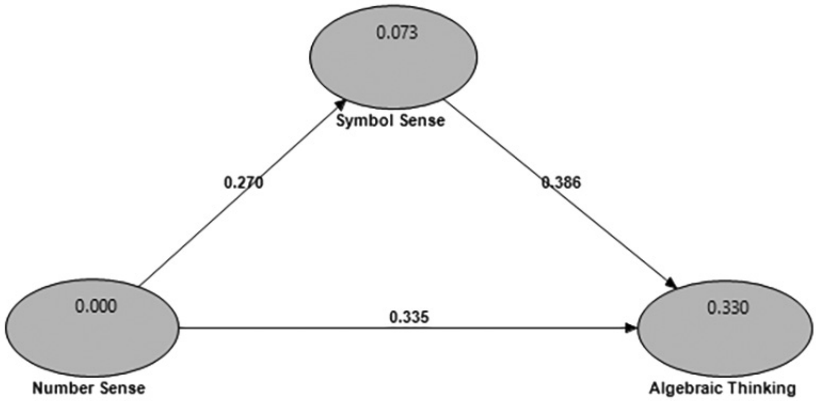

Figure 4. Mediating effect of symbol sense on the relationship between number sense and algebraic thinking

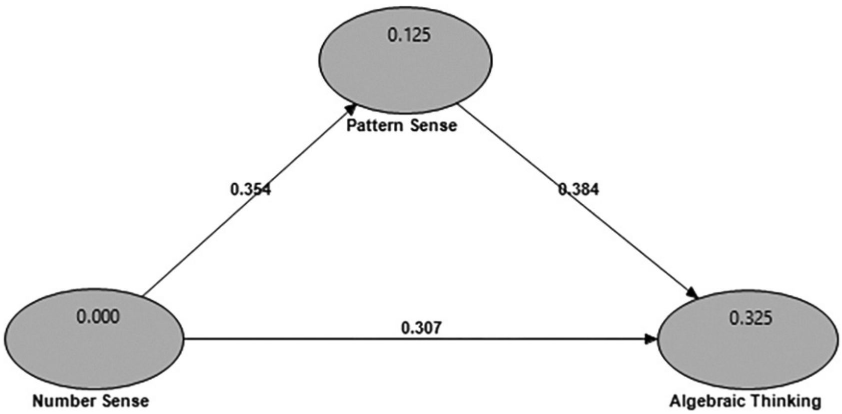

Figure 5. Mediating effect of pattern sense on the relationship between number sense and algebraic thinking

Table 1. Indirect effect, standard deviation and t-values of symbol sense and pattern sense on the relationship between number sense and algebraic thinking

Independent Variable	Mediator	Indirect Effect	Standard Deviation	*t* Value	P Values
Number Sense	Symbol Sense	0.104	0.017	6.118	< 0.001
	Pattern Sense	0.136	0.018	7.556	< 0.001

The results have shown that symbol and pattern senses have a significant mediating effect. Thus, further VAF analysis was performed to identify the mediator. Table 2 summarises the VAF values for symbol and pattern sense. About 23.7% of the number sense effect on algebraic thinking was explained via the symbol sense mediator. Similarly, 30.7% of the number sense effect on algebraic thinking was explained via the pattern sense mediator. Since VAF was larger than 20% but smaller than 80%, this situation can be considered as partial mediation.

Table 2. VAF and mediation type of symbol sense and pattern sense on the relationship between number sense and algebraic thinking

Independent Variable	Mediator	VAF Value (%)	Mediation Type
Number Sense	Symbol Sense	23.7	Partial
	Pattern Sense	30.7	Partial

Discussion

At the end of the analysis, symbol sense and pattern sense were identified as potential mediators between number sense and algebraic thinking. This shows that knowledge in symbol sense mediates how number sense could be used while working with algebraic thinking tasks. With regard to this, sense making of numbers contributes to an understanding of variables (symbol sense) and leads to algebraic thinking. In other words, number sense contributes to an understanding of variables and the equal sign, which leads to better performance in algebraic thinking (Jacobs et al., 2007). Similarly, pattern sense can influence symbol sense and number sense, which in turn can influence algebraic thinking. For example, working with patterns requires some knowledge of symbols involved, whether the pattern is growing or shrinking, and sense making of numbers to make a prediction of subsequent patterns or any arbitrary term of patterns (Lannin, 2005).

The results of this study have contributed to the body of literature on early algebraic thinking. The majority of previous research on early algebraic thinking was focused only on teaching experiments which look into children's ability to think in an algebraic way and evaluation of algebraic thinking (Carpenter & Levi, 2000; McNeil; 2008; Ralston, 2013; Rittle-Johnson & Alibali, 1999). This study has provided numeric evidence on the constructs that could play a crucial role in algebraic thinking development.

Conclusions

Number sense and algebraic thinking work hand in hand as foundation for algebra courses. Symbol and pattern sense are the two most influential factors that affect the relationship between number sense and algebraic thinking. This information presents a new topic for discourse and confirms the importance of symbol sense and pattern sense in algebraic thinking development. Given that number sense and algebraic thinking are closely connected, symbol sense and pattern sense act as mediators between number sense and algebraic thinking in building strong foundation for future formal algebra learning. Symbol sense and pattern sense can intensively improve the conceptual understanding of underlying properties of algebra while learning arithmetic in elementary mathematics classrooms. The items shown in Figures 1 and 2 are the best examples on how teachers could encourage number sense and algebraic thinking. They could incorporate symbol sense and pattern sense in their daily lessons. Early algebraic thinking does not need a new chapter to include in the existing syllabus. It is how to teach arithmetic that can build a conceptual understanding and develop children's ability to think deeper beyond mere computations.

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