

Nurulhuda Md Hassan,
Saemah Rahman
Malaysia

Problem Solving Skills, Metacognitive Awareness, and Mathematics Achievement: A Mediation Model

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Abstract

This study was aimed to identify the relationship between problem solving skills, metacognitive awareness, and mathematics achievement as well as to identify the role of metacognitive awareness as a mediator. This study involved a total of 333 Form Four students from ten secondary schools in Malaysia. Data were collected using questionnaires, while information about mathematics achievement was provided by the school management. Data were analyzed using the Structural Equation Modeling (SEM) technique. Results indicated that all variables correlated significantly with each other, while path analysis revealed the mediation effect of metacognitive awareness between problem solving skills and mathematics achievement. The findings suggest the importance of problem solving skills in influencing secondary school students' mathematics achievement through the development of metacognitive awareness.

Keywords: *problem solving, metacognitive awareness, mathematics achievement, structural equation modelling*

Introduction

Trends in International Mathematics and Science Study (TIMSS) as well as Program for International Student Assessment (PISA) reported poor mathematics performance among Malaysian students, especially at the secondary school level

(Mullis et al., 2012; Organization for Economic Cooperation and Development, 2010). For Trends in International Mathematics and Science Study (TIMSS), the assessment conducted in 2007 showed that Malaysia's ranking had dropped from the 10th place out of 45 countries to the 20th place out of 49 countries. The assessment carried out in 2011 also revealed the same scenario, i.e., that Malaysia's ranking had dropped to the 26th place out of 42 countries worldwide. The poor mathematics performance was also evident by considering the results of Program for International Student Assessment (PISA) conducted in 2009, which showed that Malaysia was ranked 57th out of 74 countries involved in that assessment. Those reports implied the failure of Malaysian students in reaching the minimum level required in enabling them to participate in life effectively and productively. Such failure need to be addressed since mathematics, apart from being a core subject in the school curriculum, has also become a requirement of most disciplines in higher education institutions (Esan, 2015).

Among the factors that had been found to be significant in influencing students' mathematics achievement, there are problem solving skills and metacognitive awareness. Problem solving is an important life skill (Karatas & Baki, 2013) which involves finding an appropriate response to a situation which is unique and novel to every problem solver (Esan, 2015). A problem solving process begins with the initial contact with a problem and ends when an answer is found in the light of given information (Olaniyan et al., 2015). In order to obtain a solution to a given problem, the problem solver must adopt various higher order thinking skills including analyzing, interpreting, reasoning, predicting, evaluating, and reflecting. Apart from being a means of stimulating intellectual curiosity, problem solving is also believed to be important in aiding the transfer of learning through the provision of opportunities for students to draw relationships between things they know and the learning task at hand (Esan, 2015).

One of the most important goals of contemporary education is to educate individuals who can overcome the problems that they encounter in their daily lives and social lives successfully (Selçuk et al., 2008). In other words, modern education aims at producing effective problem solvers. In the educational context, mathematics is one of the subjects that expose students to various problem solving situations. This is in line with the goal of mathematics education as stated in the reform documents prepared by the National Council of Teachers of Mathematics (NCTM 2000). According to NCTM (2000), opportunities should be given to students not only to apply and adapt various problem solving strategies, but also to monitor and reflect on the problem solving processes that they encounter. Provision of such opportunities was found to be positively related to students'

mathematics achievement (Mullis et al., 2012), while the problem solving skills developed by students may enable them to deal successfully with real life problems.

In mathematics, the problem solving heuristics developed by Polya (1945) has always been used in teaching problem solving. In his book 'How To Solve It', Polya (1945) listed four steps which are believed to be effective in helping students to solve mathematical problems. The first step, understanding a problem, involves activities such as rereading, paraphrasing, visualizing the problem, and determining the known and unknown information. The second step, devising a plan, involves the selection of an appropriate solution. The third step, implementing problem solving strategy, involves the execution of the planned strategy as a way to solve the problem. The final step, checking the solution, involves revision of the obtained answer in order to determine the extent of its accuracy with the given problem. The effectiveness of those steps in improving students' mathematics achievement was shown in previous studies (Bilgin 2005; Esan 2015; Karatas & Baki 2013; Olaniyan et al., 2015; Selçuk et al., 2008).

With regard to metacognitive awareness, review of the literature indicated that there are two components of this factor that play a vital role in improving student learning (Handel et al., 2013; Schraw & Moshman, 1995). The first component, which is known as metacognitive knowledge, is believed to help student in learning how, when, and where they can use their cognitive strategies (Breed et al., 2014). This component was further divided into three subcomponents, namely declarative knowledge, procedural knowledge, and conditional knowledge. Declarative knowledge refers to knowledge of self, tasks, and strategies that are relevant to the task; procedural knowledge refers to knowledge about how to implement a strategy; whereas conditional knowledge refers to knowledge about when and why certain strategies are useful at certain times (Lee & Schmitt, 2014). In the educational context, metacognitive knowledge refers to what students know about how they learn, what students know about the procedures and strategies that are most effective for them, and what students know about the conditions under which various cognitive activities are most effective to be used.

The second component, known as regulation of cognition, refers to the actual activities engaged by students in order to facilitate their learning (Young & Fry, 2008). This component involves processes through which students think about their thinking in order to develop proactive strategies to solve a particular task (Bonner, 2013) and consists of three higher level thinking skills which are believed to be important in ensuring the effectiveness of one's learning processes, namely planning, monitoring, and evaluation (Tavakoli, 2014). In the educational context, planning involves students' effort to plan out a particular cognitive task by selecting

appropriate strategies and cognitive resources. Monitoring involves the awareness of their progress through a cognitive task and students' ability in determining their own performance. Evaluation involves students' effort to take a look at the outcome and determining the extent to which the learning outcome matches their learning goals as well as the effectiveness of their regulation processes (Schraw & Moshman, 1995; Young & Fry, 2008).

Previous studies not only provided evidence regarding the role of problem solving skills and metacognitive awareness in influencing student achievement (Adeoye, 2010; Bilgin, 2005; Esan, 2015; Karatas & Baki, 2013; Olaniyan et al., 2015; Selçuk et al., 2008; Areepattamannil & Caleon, 2013; Tavakoli, 2014; Young & Fry, 2008), but also revealed the role of problem solving skills in influencing the development of metacognitive awareness (Breed et al., 2014; Lee & Schmitt, 2014; Liu et al., 2012; Nasarudin Abdullah et al., 2014; Osman, 2010; Shen & Liu, 2011; Yildirim & Ersözülü, 2013). According to Yildirim and Ersözülü (2013), problem solving requires one's engagement in various metacognitive processes such as planning and evaluating. In this regard, problem solving is considered as not only involving one's effort to find the right solution, but also requiring an understanding and mastery of higher order thinking skills such as planning, monitoring, and evaluating (Nasarudin Abdullah et al., 2014). In other words, metacognitive aspects are believed to be a prerequisite for effective problem solving. Thus, the teaching of problem solving in mathematics should emphasize the involvement of higher order thinking skills including metacognitive awareness while students engage in problem solving processes.

Based on the literature review, it is crucial to examine whether or not problem solving skills and metacognitive awareness are significantly associated with mathematics achievement among secondary school students. Besides, the growing evidence which showed the role of problem solving skills in supporting the development of metacognitive awareness may also be linked to students' mathematics achievement. Hence, this research aimed to investigate the relationship and mediation effect between problem solving skills, metacognitive awareness, and students' mathematics achievement. Specifically, the objective of this study was to investigate the role of metacognitive awareness as a mediator in the relationship between problem solving skills and secondary school students' mathematics achievement. In this study, it was hypothesized that problem solving skills and metacognitive awareness would be significantly and positively associated with mathematics achievement. Besides, it was also hypothesized that metacognitive awareness mediates the relationship between problem solving skills and mathematics achievement.

Research Methodology

This study involved a total of 333 Form Four students from ten secondary schools located in one of the states in Malaysia, who had been selected by using multistage stratified random sampling. This study was an ex post facto study which employed a survey method. Three instruments were used for the purpose of data collection in this study:

a) Problem Solving Skills

A questionnaire, which consists of 28 items, was developed in order to gather information about the students' problem solving skills. Twenty of the items were adapted from Kiong (2005), while the remaining items were developed by the researcher. The items were related to the four problem solving steps as proposed by Polya (1945), namely understanding a problem, devising a plan, implementing a problem solving strategy, and checking the solution. Results of factor analyses had been used in determining the validity and reliability of this instrument. Exploratory factor analysis (EFA) extracted four factors which were in line with the four problem solving components, while confirmatory factor analysis (CFA) revealed that the obtained Average Variance Extracted (AVE) and construct reliability (c.r) were 0.612 and 0.861, respectively.

b) Metacognitive Awareness

The Metacognitive Awareness Inventory, developed by Schraw and Dennison (1994), was used to gather information about the students' metacognitive awareness. The items were categorized according to the six subcomponents of metacognitive awareness as suggested by Schraw and Moshman (1995), namely declarative knowledge, procedural knowledge, conditional knowledge, planning, monitoring, and evaluation. The validity, reliability, and suitability of using this inventory in the Malaysian context had been reported by previous study (Sendurur et al., 2011).

c) Mathematics Achievement

The scores obtained by the students in three mathematics assessments were used to gather information about the students' mathematics achievement. Specifically, those assessments include a monthly test, final year examination, and Program for International Student Assessment (PISA). The scores were provided by the school management and had been standardized in order to ensure their uniformity.

Structural equation modeling (SEM) through Analysis of Moment Structures (AMOS) software was used to test the study hypotheses. In this regard, two

structural models were analyzed. The first structural model involves analysis of the direct effect between metacognitive awareness and mathematics achievement, while the second structural model entails analysis of the mediation model.

Research Results

Analysis of the direct effect between problem solving skills and mathematics achievement indicated that the obtained beta coefficient value was 0.692 at $p < 0.001$. This result suggests that problem solving skills had a positive, significant direct effect on the secondary school students’ mathematics achievement. The obtained value of coefficient determination (R^2) was 0.48, which indicated that problem solving skills contributed to 48% of variance in mathematics achievement. The results of the first analysis are shown in Figure 1.

Analysis of the mediation model indicated that problem solving skills still had a positive, significant direct effect on mathematics achievement when metacognitive awareness was added as a mediator, but the value of the effect decreased to 0.488. Such a decrease provided evidence that metacognitive awareness mediated the relationship between problem solving skills and the secondary school students’ mathematics achievement. The obtained value of coefficient determination (R^2) was 0.51, which indicated that 51% of variance in mathematics achievement was explained by problem solving skills and metacognitive awareness collectively. The results of the second analysis are shown in Figure 2.

The results of the relationship among variables in this study from both analyses are shown in Table 1.

Table 1. Relationships among variables

Variables		β	P
Direct Model			
Problem Solving Skills	————>	Mathematics Achievement	0.692 <0.001
Mediation Model			
Problem Solving Skills	————>	Mathematics Achievement	0.488 <0.001
Problem Solving Skills	————>	Metacognitive Awareness	0.769 <0.001
Metacognitive Awareness	————>	Mathematics Achievement	0.270 <0.001

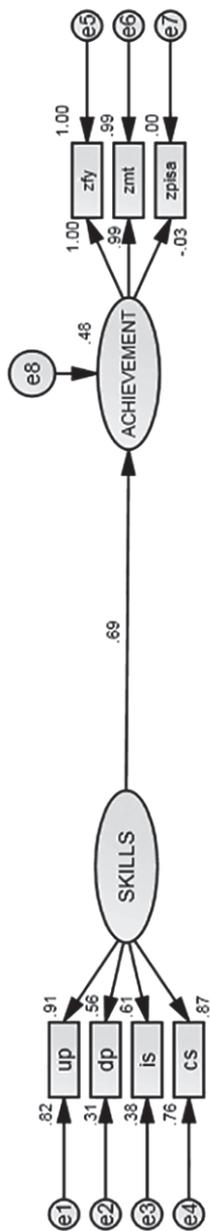


Figure 1. Direct effect of problem solving skills on mathematics achievement

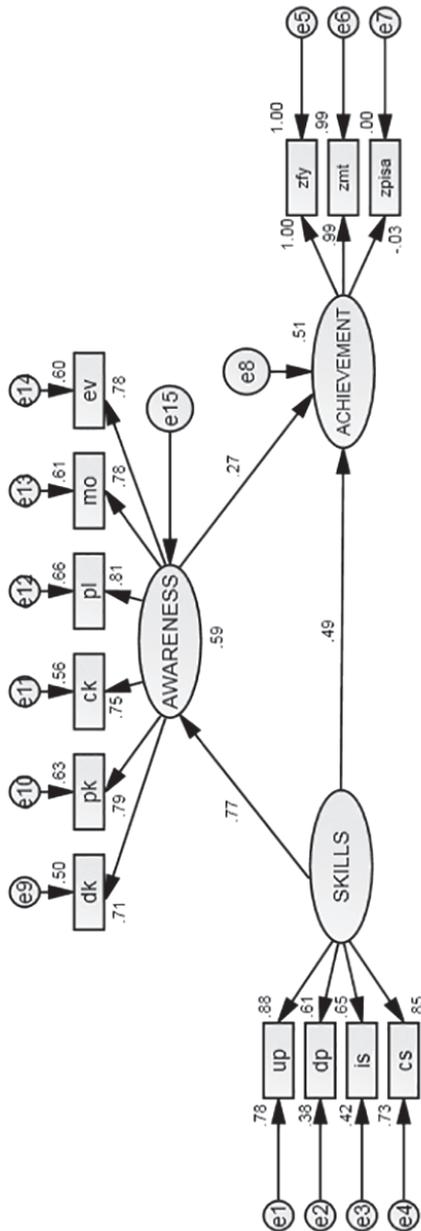


Figure 2. Analysis of mediation model

Discussion

Based on the results, problem solving skills were found to be positively associated with mathematics achievement. This is in line with the findings reported by previous studies (Bilgin, 2005; Esan, 2015; Karatas & Baki, 2013; Nasarudin Abdullah et al., 2014; Olaniyan et al., 2015; Selçuk et al., 2008; Yetik et al., 2012). This finding can be justified by taking into account the role of problem-solving skills in improving the students' mathematics achievement. According to Yetik et al. (2012), students' problem solving skills may influence their involvement in problem solving steps when trying to solve mathematical problems. In other words, those skills encourage students to get involved in problem solving activities such as understanding the problem, devising a plan, implementing a problem solving strategy, and revising a solution. According to Nasarudin Abdullah et al. (2014), problem-solving skills allow students to strengthen their conceptual understanding, procedural fluency, strategic competence, productive disposition, and adaptive reasoning abilities. The application of such aspects, particularly in the context of assessment, is believed to directly contribute to an increase in student achievement.

The results of this study also showed a positive, significant relationship between metacognitive awareness and mathematics achievement. This is consistent with the findings reported by previous studies (Arrepattamannil & Caleon, 2013; Hong-Nam, 2014; Sperling et al., 2012; Tavakoli, 2014; Young & Fry, 2008). This finding can be justified by considering the role of metacognitive awareness in improving student achievement. According to Sperling et al. (2012), most researchers agreed that metacognitive awareness plays an important role in influencing student learning and achievement. Young and Fry (2008) stated that all components and subcomponents of metacognitive awareness involve aspects that can help students improve their achievement. For instance, conditional knowledge allows students to learn appropriate conditions for the implementation of a learning strategy, while planning allows students to choose strategies that can be used to accomplish a particular task. In this regard, Young and Fry (2008) claimed that students who apply metacognitive aspects are believed to have outstanding academic achievement. Hong-Nam (2014) pointed out that metacognitive awareness can encourage students to apply certain strategies in order to complete a task, which in turn had a positive impact on their achievement. This statement was supported by Arrepattamannil and Caleon (2013), who claimed that students who have metacognitive awareness use their skills to select an appropriate approach to achieve their learning goals. According to them, students who have the skills to control their

learning processes are able to identify learning objectives to be achieved, which in turn encourages them to make a choice regarding the appropriate approach to achieve those objectives.

In addition, the results of this study also indicated a positive, significant relationship between problem solving skills and metacognitive awareness. This finding is consistent with the findings from previous studies (Gultepe et al., 2013); Nasarudin Abdullah et al., 2014; Osman, 2010; Roslina, et al., 2010; Yildirim & Ersözlü, 2013). This finding can be justified by taking into account the role played by metacognitive awareness in ensuring the success of problem solving process. According to Yildirim and Ersözlü (2013), problem solving requires students' involvement in cognitive and metacognitive processes. To illustrate, problem solving processes require the selection of a particular strategy as well as consideration of alternative strategies, whereby both manifest the cognitive processes encountered by students. Such cognitive processes should be supported by metacognitive processes such as monitoring, which will control and monitor the effectiveness of those cognitive processes. Nasarudin Abdullah et al. (2014) also supports this statement by claiming that the complexity of problem solving processes require students to integrate some cognitive and metacognitive elements in order to find the right solution. According to them, besides involving an attempt to get the right answer, problem solving also involves understanding and control of more complex metacognitive strategies, such as planning, monitoring, and evaluation. Specifically, Nasarudin Abdullah et al. (2014) argued that self-regulation skills, which include planning, monitoring, and evaluation, will help students in developing the necessary skills to solve mathematical problems. According to Roslina et al. (2010), metacognitive awareness allows students to realize their thought processes when solving mathematical problems. Besides having knowledge of facts and procedures, students who have metacognitive awareness also have the knowledge about how and when to use the facts and procedures effectively. In addition, regulation of cognition enables students to control, monitor, and evaluate the progress of problem solving processes effectively.

Besides, the results also showed that metacognitive awareness mediated the relationship between problem solving skills and mathematics achievement. This result indicated that problem solving skills may influence mathematics achievement through metacognitive awareness. This finding can be justified by taking into account the role of metacognitive awareness in influencing problem solving skills, which in turn leads to an increase in student achievement. According to Hong-Nam (2014), metacognitive awareness helps students to reflect on their thinking process, in which the reflection occurs in three phases, i.e., before, during, and

after the problem solving process. More specifically, metacognitive awareness encourages students to plan, monitor, and evaluate the effectiveness of the problem solving process. Hong-Nam (2014) also stated that cognitive and metacognitive actions taken by students during the problem solving process allow students to enhance their learning. He claimed that when students have metacognitive awareness, they are able to apply their knowledge and skills effectively when solving a mathematical problem, including when dealing with mathematics assessment that usually involves problem solving. This in turn helps in contributing to the improvement of students' mathematics achievement.

Conclusions

The findings of this study have several important educational implications, especially in the secondary school context. Based on the findings, it is clear that both problem solving skills and metacognitive awareness play an important role in improving secondary school students' mathematics achievement. Therefore, attention should be paid to the development of problem solving skills and metacognitive awareness in order to improve students' mathematics achievement. Apart from that, this study also provided evidence regarding the role of metacognitive awareness as a mediator in the relationship between problem solving skills and mathematics achievement. This finding indicated that the development of problem solving skills can be enhanced when students possess metacognitive awareness, which consequently leads to higher student achievement. Thus, it is very important to help students develop both metacognitive awareness and problem solving skills since both factors were found to be related to mathematics achievement. Based on these findings, teachers are recommended to integrate the elements of metacognitive awareness in developing students' problem solving skills in order to help them become metacognitive experts, who are aware of their thinking processes and have self-regulated learning skills, which consequently leads to the improvement of their mathematics achievement.

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